UNDERSTANDING SUB-HARMONICS

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Introduction

Over the years, engineers have employed fundamental principles of electrical engineering to solve system problems. For example, series reactors are used to limit the amount of fault current since inductors resist a change in current. Similarly, series line capacitors are used to increase the amount of real power by cancelling a portion of the inductive reactance. However, some solutions can create new problems in the system, as is the case with series line compensated capacitors. When a capacitor is used in a transmission line as a series element, sub-harmonic currents with frequencies below the fundamental are created. Extensive research and study have been done, and the consequences of sub-harmonic currents can be catastrophic.

The effects of sub-harmonics were visualized back in 1971 after a synchronous generator shaft failure [3]. Since then, generator manufacturers and utilities have worked together to identify problems between the interaction of series capacitors and generators. In recent months, a couple of incidents—one in Texas and another in Minnesota—have been identified where sub-harmonics were implicated in creating an increased voltage and current oscillations. One common factor in these two events is that they both involved wind generation.

After decades of research, there are various phenomena that can be attributed to sub-harmonics. Sub-harmonics can create induction generator effects, torsional interactions, torque amplification, sub-synchronous resonance, and transformer saturation. For the interaction between wind generation and series compensated lines, research that will help determine the real cause is still underway; however, the evidence records seem to point towards an induction generator effect phenomena. As a result, this paper provides focuses the induction generation effect, and gives a brief description of torsional interactions and torque amplifications. Additionally, this paper provides simulations that help the reader to visualize the effect and offers a relay solution for sub-harmonics.

Sub-Harmonics

It is well known that nonlinear loads will produce harmonics by drawing currents that are not necessarily sinusoidal. In effect, inductive loads will produce harmonics that are multiples of the fundamental (e.g., 120Hz, 180Hz, 240Hz, etc.) In a similar manner, when a circuit involving a resistor, capacitor and inductor is connected in series, voltages and currents with frequencies below the fundamental (e.g., 20Hz, 25Hz, 30Hz, etc.) will be created [2]. These are called sub-harmonic frequencies; they will be denoted *fer*. Sub-harmonics were first discussed by Butler and Concordia shown in reference [4]. Experimenting with series capacitors and transformers in series, they discovered a large, abnormal current flow that was low in frequency and had distorted the entire current waveform. In addition, the transformer secondary voltage was also distorted.

Subsequent research was performed, and the researchers discovered that the system resistance played an important role in the magnitudes of these sub-harmonic currents. It is now known that the sub-harmonic frequencies in an RLC circuit will be damped out through the resistance of the circuit and will coexist in the system without presenting major problems [5]. However, during faults or switching events, the sub-harmonic currents are amplified and excited so that the resistance of the circuit can reach a point that is not enough to damp the sub-harmonic frequencies [6]. As a result, these frequencies can cause voltage and current amplification, as explained by the induction generator effect.

Induction Generator Effect

When a series capacitor is used to cancel a portion of the system reactance, the system will always end up with a natural frequency which is less than the system frequency and is referred to as the sub-harmonic [2]. This sub-harmonic frequency is defined by as:

$$fer = fo * \sqrt{\frac{Xc}{(X" + \ Xe + Xt)}} \quad (1)$$

The generator armature sub-harmonic currents produce magnetic fields with frequency *fer* which are induced in the generator rotor. As a result, the induce currents in the rotor produces a rotor frequency defined as:

$$fr = fo - fer \quad (2)$$

In consequence, this new added rotor frequency will result in sub-synchronous armature voltages which may enhance the sub-synchronous currents and may cause generator self–excitation [1].

Because the rotor is turning faster than the sub-harmonic armature currents with frequency *fer*, a slip is created simulating an induction machine and the slip is defined by s:

$$Slip(s) = \frac{fer - fo}{fer} \quad (3)$$

As we can see from equation 3, the slip will always be negative, which will result in a negative rotor resistance R' as described in Figure 1.

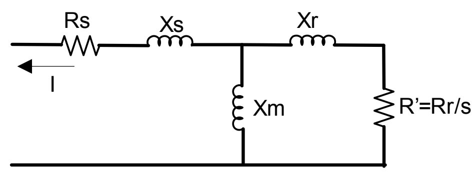

Figure 1: Induction generator effect circuit.

The rotor resistance is defined as:

$$R' = \frac{Rr}{s} \quad (4)$$

Since the slip (s) will always be negative, the damping effect of *R'* will always be negative. "If the series compensation is very high, the slip (s) will be very small and therefore R' will be a negative large number" [6]. If the addition of the damping resistance *R'* and the resistance of the system is negative, voltage and current growing oscillations may build up to very dangerous high values. In order to emphasize this concept better, let take a look at the following example.

Figure 2 shows a series compensation circuit through a couple 525kV transmission line given by reference [7].

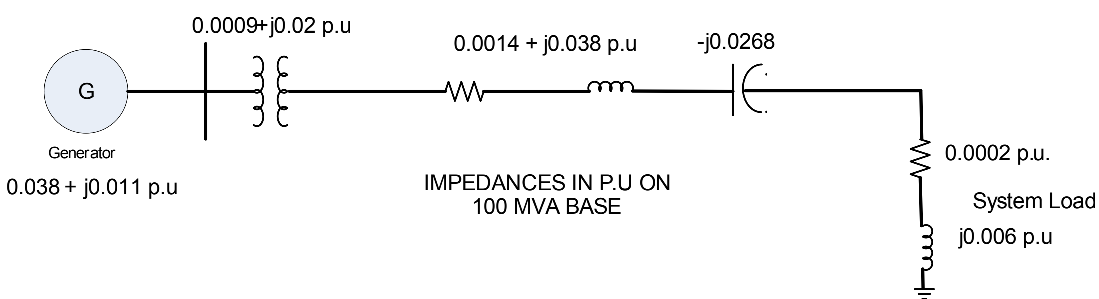

Figure 2: Series Compensation Circuit

Using equation 1 we can find the natural frequency of this circuit as follows:

$$fer = fo * \sqrt{\frac{Xc}{(X" + \ Xe + Xt)}}$$

$$fer = 60 * \sqrt{\frac{0.0268}{0.075}}$$

$$fer = 36Hz$$

Using equation 3 we can calculate the slip:

$$Slip = \frac{fer - fo}{fer}$$

$$Slip = \frac{36 - 60}{36}$$

$$Slip = \ -0.66 \text{ p.u.}$$

Using equation 4 we can calculate the damping with a rotor resistance of 0.02 p.u:

$$R' = \frac{R}{s}$$

$$R' = \frac{0.0038}{-0.66}$$

R' = -0.0057 p.u.

The total resistance of the network of line, transformer and load is:

Rnetwork = 0.002 p.u.

The total effective resistance is:

Reff = R' + Rnetwork

Rnet = -0.0057 + 0.002 p.u.

Rnet = -0.0037 p.u.

Since the net resistance is negative for this series compensated circuit, we can expect increasing sub-harmonic oscillation.

Power System Model

The simulations in **Figure 2** below show the impact of the induction generator effects for the system.

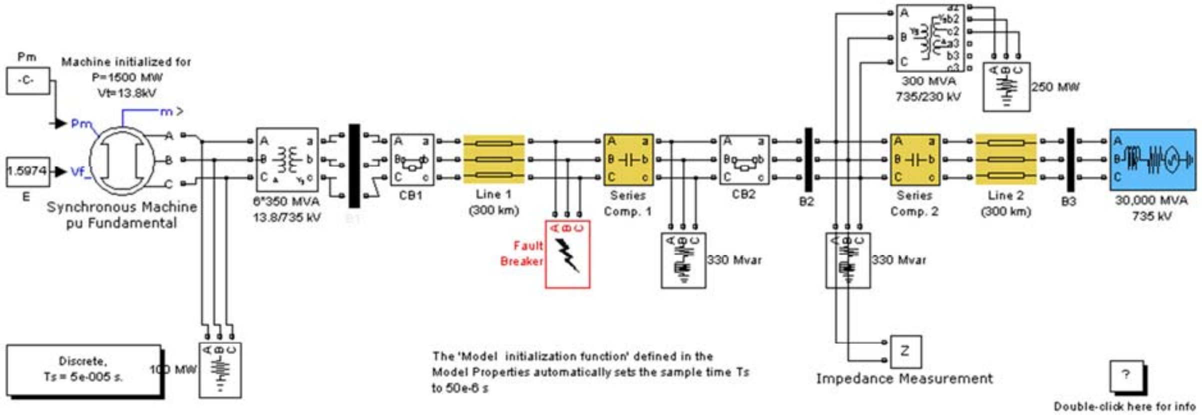

Figure 2: Three phase series compensated system.

Figure 2 shows a three phase series compensated line composed of six 350 MVA generators, a 13.8 to 735kV step-up transformer, two 300 km 735kV transmission lines, and two 68.2uf capacitors for 40% line compensation [8].

Case 1: System Response to Sub-Harmonics with Line to Ground Fault.
The model takes into account the dynamics of the stator, field, and damper windings. The equivalent circuit of the model is represented in the rotor reference frame.

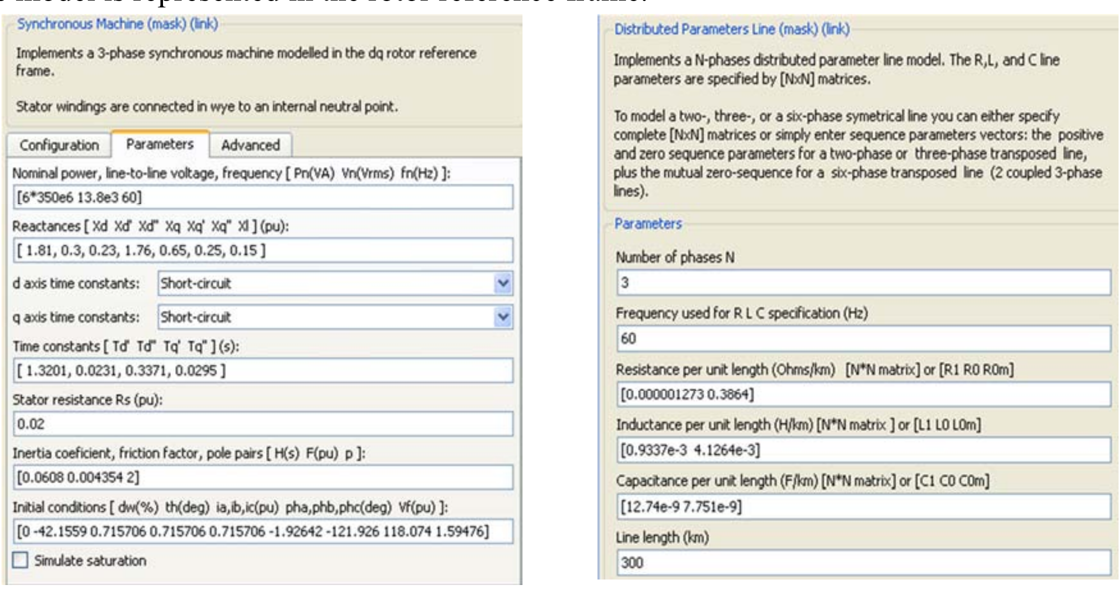

Figure 3: Generator and line parameters.

Figure 3 shows the synchronous machine and line parameters used for this simulation. For this machine model, all the stator and rotor resistances are viewed from the stator. The rotor is modeled as round type. Since the rotor resistance is dependent on the speed of the machine as Rr/s, it is evident that the rotor resistance will vary. If the net resistance (the addition of the rotor resistance and the resistance of the network) is negative, then we can expect growing oscillations.

A single line-to-ground fault is applied in the transmission line. The fault lasts for 3 cycles and then goes away. **Figure 4** shows the voltage and current response to the fault as seen from bus 1. It can be observed that the faulted voltage phase goes down and its respective current goes up. The voltages seem to recover to a fairly steady state condition. However, we can see that the currents do not. The shape of the currents signals the presence of other frequencies besides the fundamental. A frequency analysis tool can reveal embedded frequencies in the system. **Figure 5** shows the presence of the highest sub-harmonic frequency at 12Hz.
Nevertheless, the currents seem stable, and growing oscillations are prevented. This means that the net resistance is positive and keeps the current and voltage from growing oscillations. The currents and voltages have a similar behavior at bus 2, as indicated in **Figure 6**.

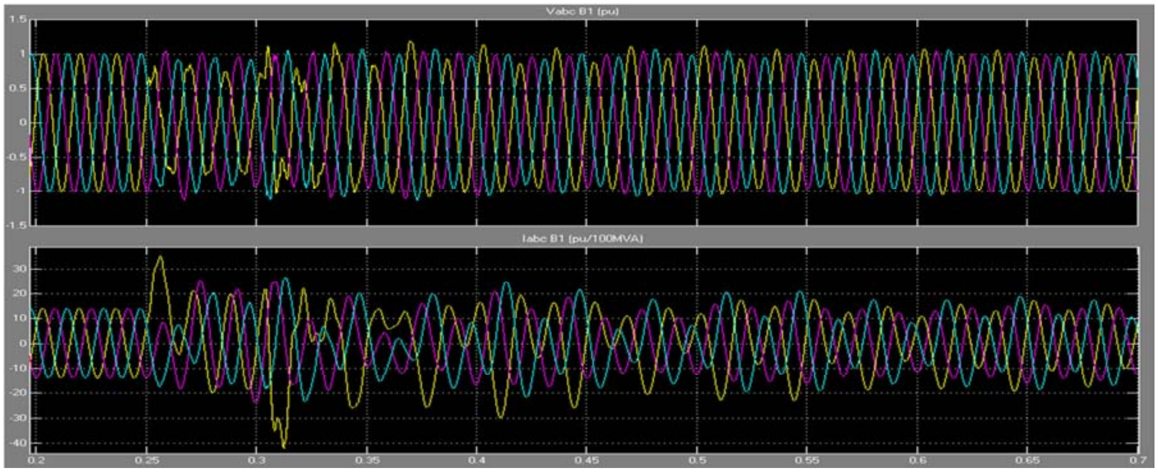

Figure 4: Line to ground fault, as seen from bus 1.

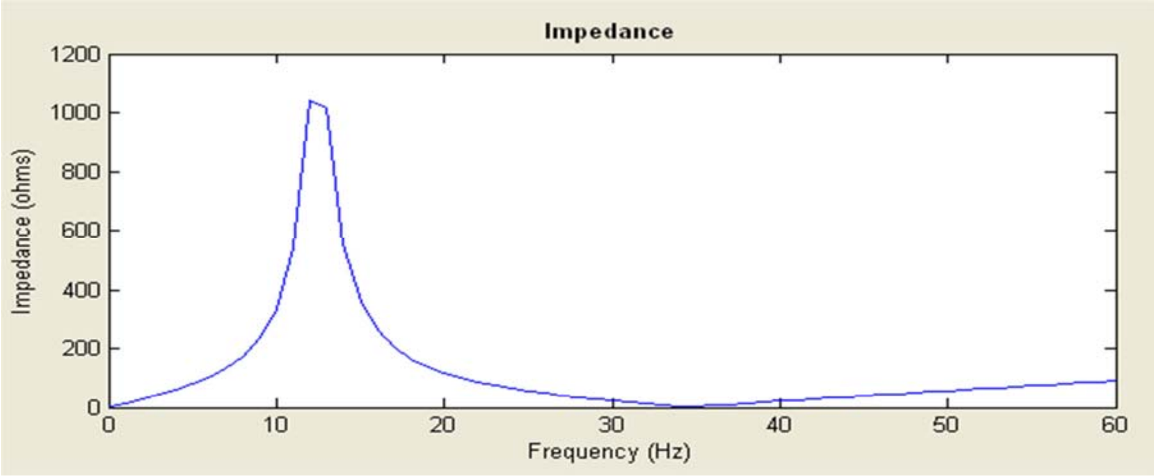

Figure 5: Sub-harmonic frequency at 12Hz.

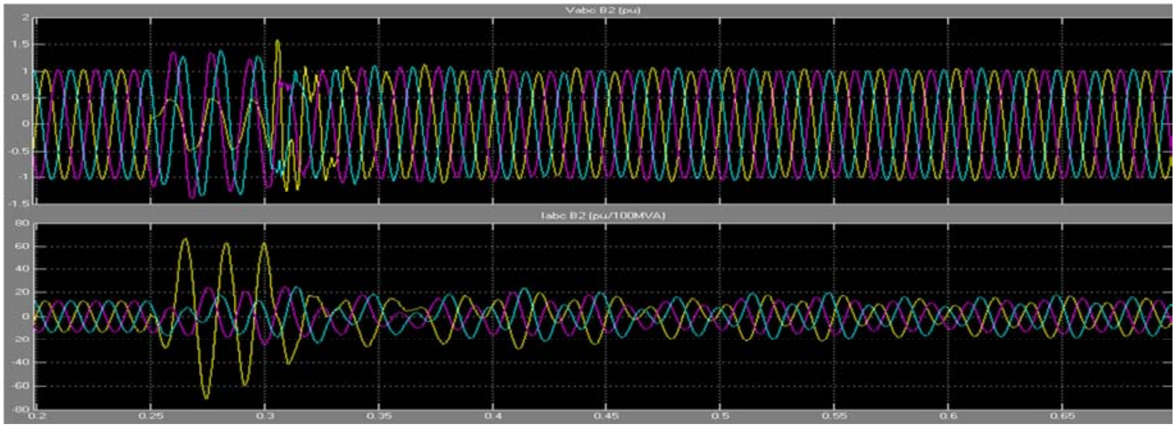

Figure 6: Line to ground fault, as seen from bus 2.

Case 2: System Response to Sub-Harmonics with Three Phase to Ground Fault.
For this second case, a three phase fault is applied to the first transmission line. **Figure 7** shows the current and voltage responses as seen from bus 1. The voltage waveform shows a severe presence of sub-harmonic frequencies. The highest of the sub-harmonic frequencies is found at 12Hz. However, due to the three phase fault, the sub-harmonic contribution is seen in all three phases. We can see that the resistance of the system is not enough to damp out the sub-harmonics, and severe voltage growing oscillations are observed. As a result, the net resistance between the rotor resistance and system resistance becomes negative and produces a negative damping effect.

An interesting observation is that the voltage magnitude reaches a value of three per unit 45 cycles after the fault inception. Such high magnitude voltage values can be avoided by an overvoltage setting. If

such growing oscillations are not stopped, equipment insulation could fail catastrophically. **Figure 8** shows the similar result for the voltage and currents as seen at bus 2.

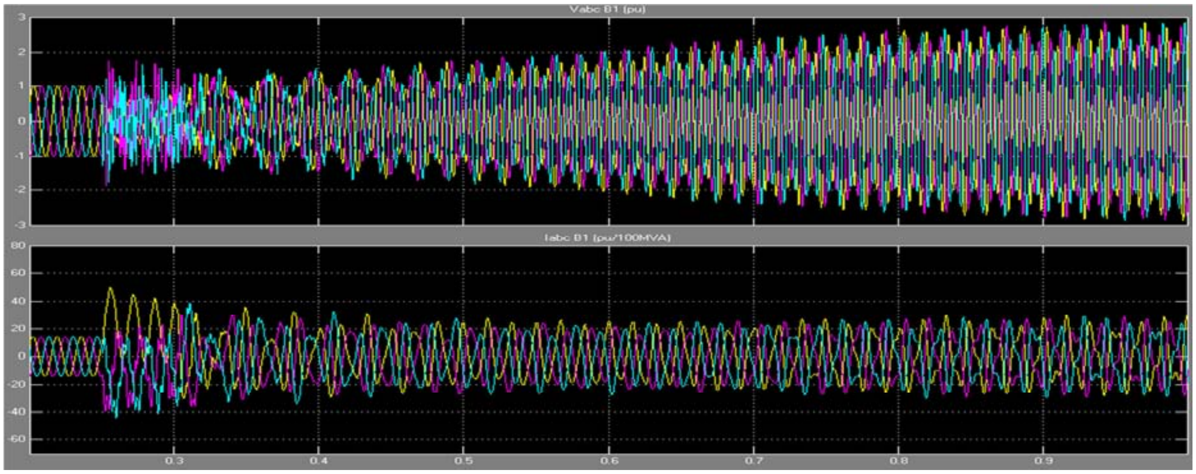

Figure 7: Three phase fault, as seen from bus 1.

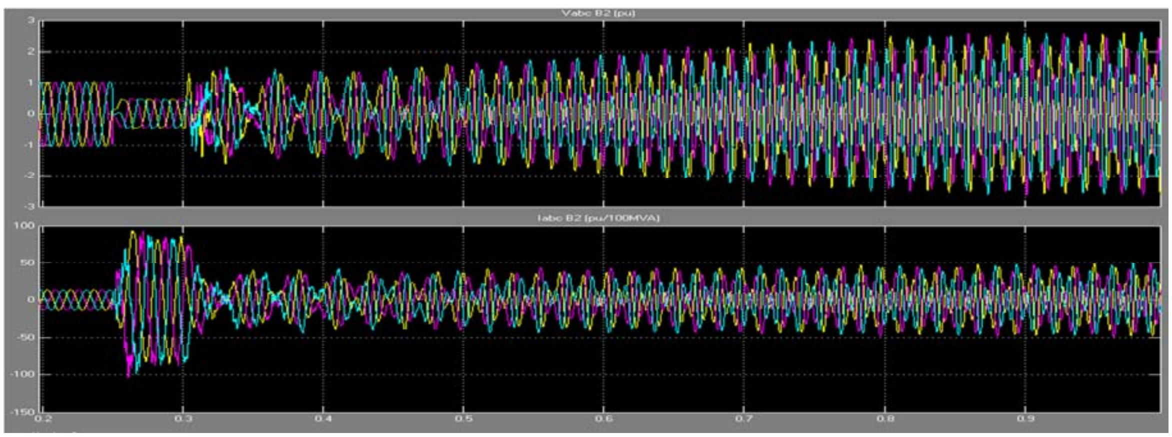

Figure 8: Three phase fault, as seen from bus 2.

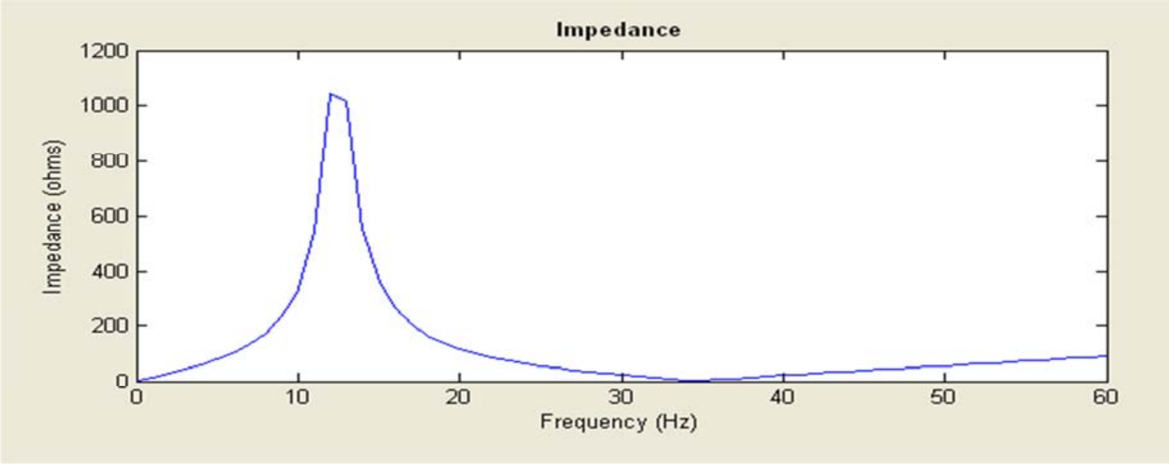

Figure 9: Sub-harmonic frequency at 12Hz.

The Effect of Net Resistance on Sub-Harmonics.
The induction generator effect occurs when the resistance Rr/s is greater than the resistance of the system, producing a negative net resistance. As a result, the negative resistance has a negative damping effect, which produces growing oscillations. In order to prove this concept, we can increase the line resistance until the net resistance becomes positive. **Figures 10** and **11** show how the growing oscillations were contained when the line resistance was increased from 0.000001273 p.u. to 1.000001273 p.u. The presence of sub-harmonics is still evident, but the net resistance is positive and damped out.

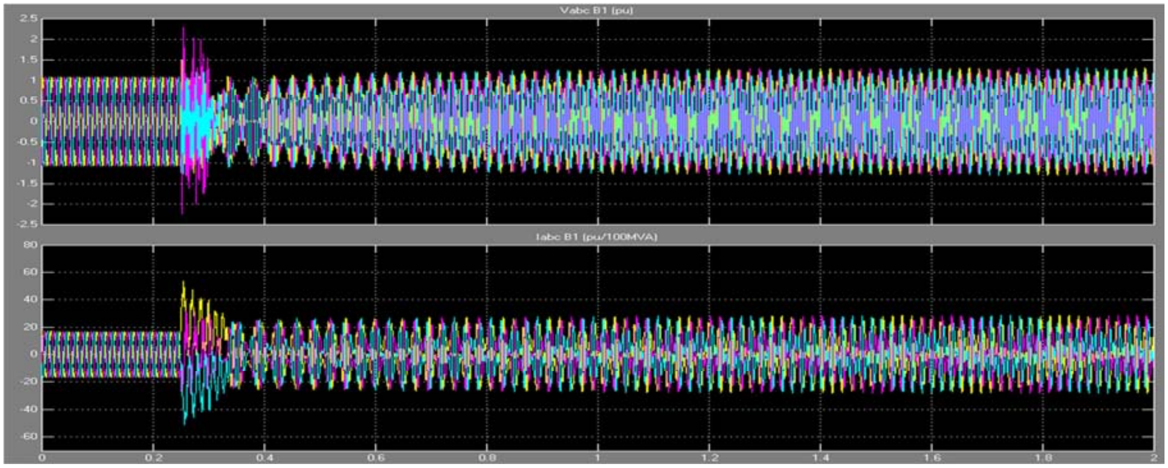

Figure 10: Three phase fault, as seen from bus 1.

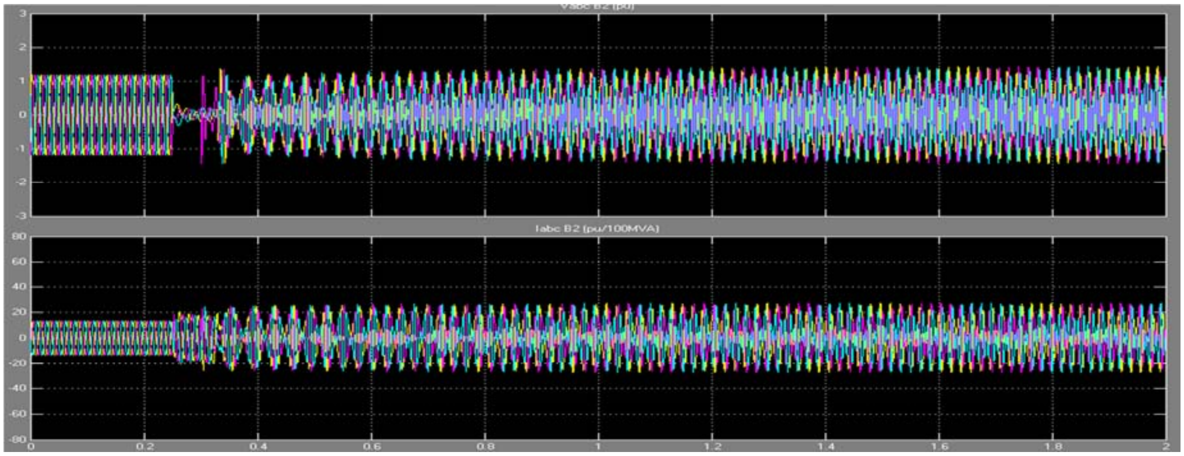

Figure 11: Three phase fault, as seen from bus 2.

Torsional Interactions and Torque Amplification

The positive sequence sub-harmonic armature currents produce rotor sub-harmonic currents at frequency *fr* = *fo*-*fer*. So we would expect this current frequencies to be less than the system current frequency of 60Hz. For our last example, the rotor sub-harmonic frequency would be:

$$fr = fo - fer$$

$$fr = 60 - 36$$

$$fr = 24\ Hz$$

If this rotor frequency coincides with the mechanical shaft system frequency *fn* (which is described below), the shaft torque could be amplified due to the resonance between the electrical and mechanical natural frequencies (1).

The effects of power system faults and switching in a series compensated circuit excites not only the electrical natural frequency modes or sub-harmonics, but also the mechanical sub-harmonics modes. A mechanical system is modeled below by 3-masses: mass 1 = generator; mass 2 = low pressure turbine (LP); mass 3 = high pressure turbine (HP).The mechanical sub-harmonic natural frequency is given by:

$$fn = \sqrt{\frac{K12}{\frac{M1 * M2}{M1 + M2}}}$$

When the rotor oscillates at frequency *fn*, it will modulate the generator voltage at a frequency *fen* which is:

$$fen = fo - fn$$

When this frequency is near the *fer* frequency, the armature current will produce torque which can create growing oscillations (1).

Figure 12 shows how the torques from the generator and low pressure turbine have been amplified to levels up to 4 p.u. due to the resonance effect.

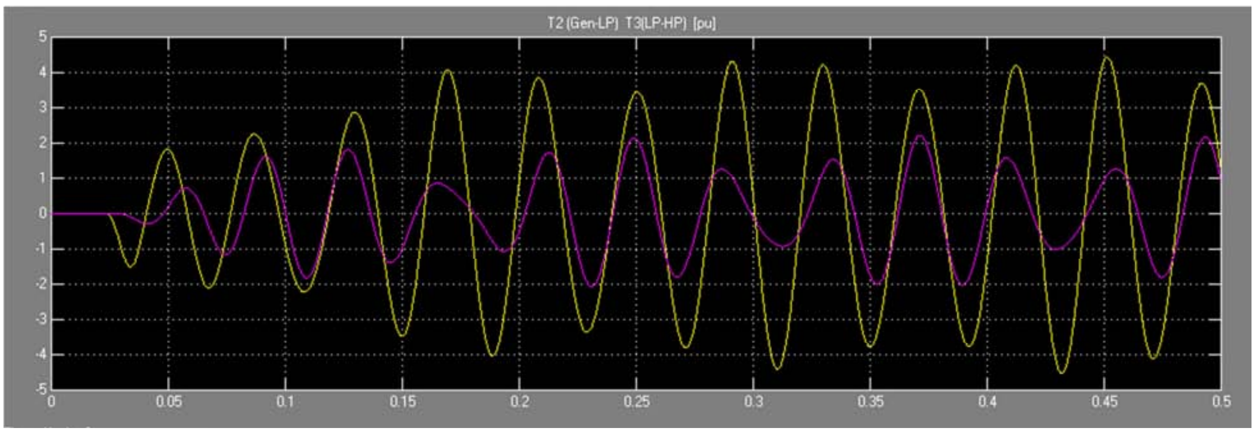

Figure 12: Generator and low pressure turbine torque due to resonance effect

Solutions to the Sub-Harmonic Problem

Obviously, varying the system resistance is not feasible, and therefore, other forms of prevention and protection are necessary. A sub-harmonic protective relay can be used to detect the sub-harmonic frequencies and allow the relay to take corrective action.

The new S-PRO relay manufactured by ERLPhase Power Technologies offers the user a comprehensive sub-harmonic solution. The S-PRO offers 5-45Hz frequency protection, under-overvoltage protection, and overcurrent protection. **Figure 12** shows the relay oneline that the S-PRO offers.

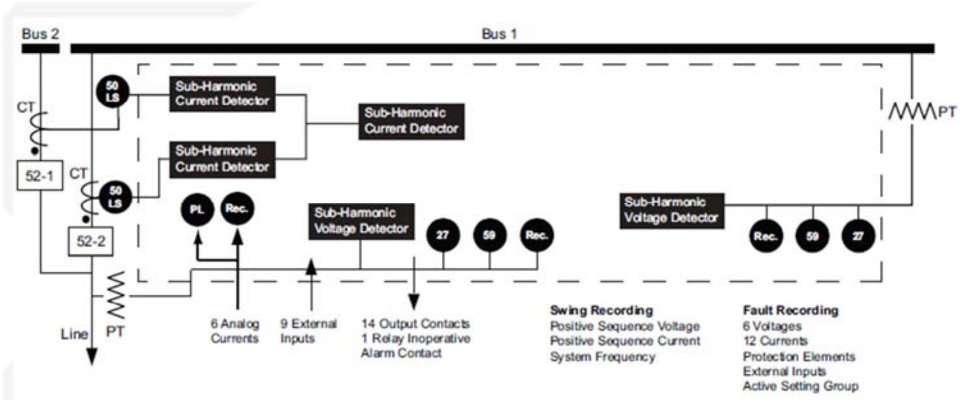

Figure 12: S-PRO relay oneline diagram.

In addition to relay protection, the S-PRO offers a very useful oscillographic program that allows the relay engineer to graph the voltages and currents along with the frequency spectrum. As a result, the engineer can quickly discover which sub-harmonic frequency is present in the system. **Figures 13** and **14** show the benefits of the oscillographic program taken from a real system event.

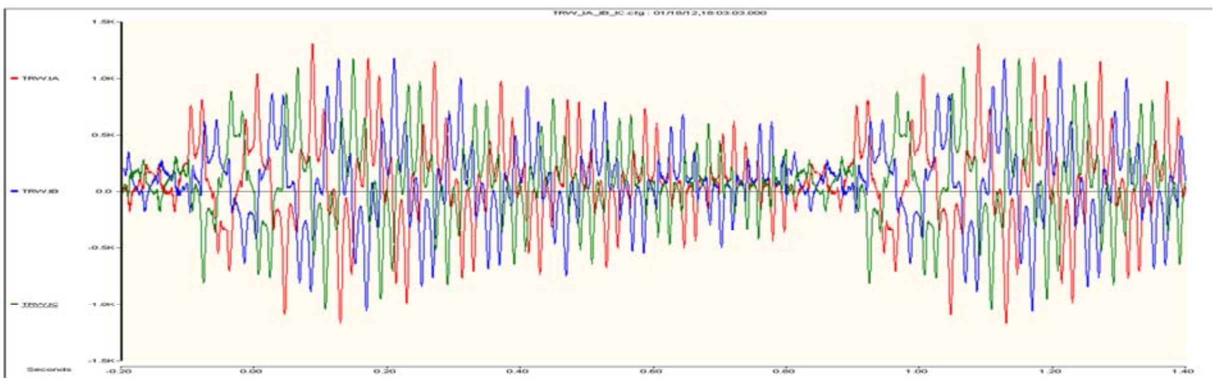

Figure 13: Sub-harmonic currents and voltages from a real event.

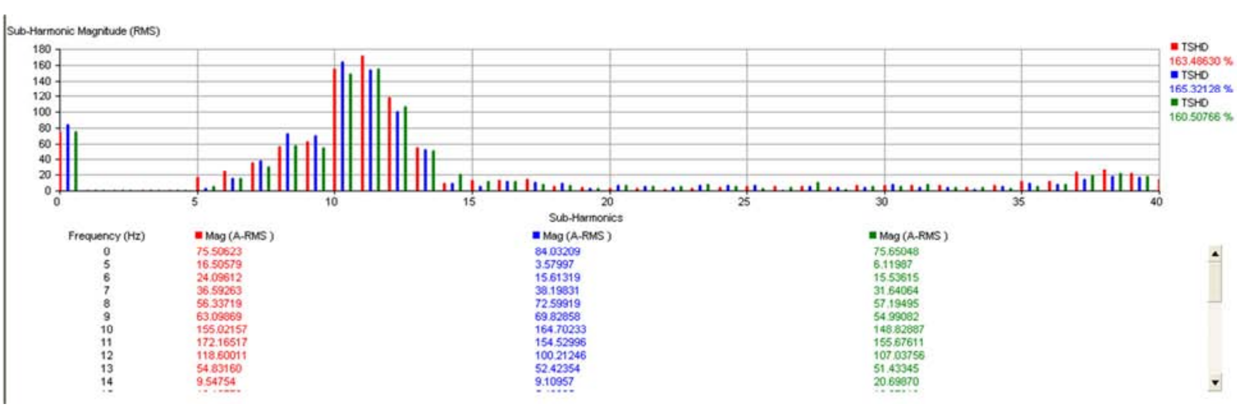

Figure 14: Sub-harmonic spectrum from a real event

References

[1] Institute of Electrical and Electronic Engineers. 1992. Reader's Guide to Subsynchronous Resonance. IEEE Committee Report. IEEE Transsactions on Power Systems, Vol 7, 150-157.

[2] Kilgore L. A. , Elliott L. C. , Taylor E. R. (1971). The prediction and control of self-excited oscillation due to series capacitor in power system. *IEEE Transactions on Power Apparatus and Systems*, PAS – 90, 1305-1311.

[3] Farmer R.G, Schwalb A.L. (1977). Navajo project report on subsynchronous resonance analysis and solutions *IEEE Transactions on Power Apparatus and Systems,* 96, 1226 – 1232.

[4] Butler J. W., Concordia C. (1956). Analysis of series capacitor application problems. *Transactions of the American Institute of Electrical Engineers*, 56, 975 – 988.

[5] The intuitor. Retrieved December 2011, from http://www.intuitor.com/resonance/abcRes.html

[6] Wadhwa C. L. (2005). Electric power systems. New Delhi, India: New Age International.

[7] Ballance, J.W., Goldberg, S. (1973). Subsynchronous resonance in series compensated transmission lines. *Transactions of the American Institute of Electrical Engineers,* PAS-92, 1649 – 1658.

[8] The MathWorks INC. (2003). SimPowerSystems. Natick, MA: Author.

Biography

Joe Perez is an Application Engineer for the relay and digital fault recorder products of ERLPhase Power Technologies, formerly NXTPhase T&D. He previously worked as a transmission engineer and a field application engineer, gaining experience in system protection projects and transmission system studies, including fault, power flow, and contingency analysis. Joe graduated from Texas A&M University in 2003 with a BSEE and has previously presented papers at WPRC, Texas A&M and Georgia Tech Relay Conferences. He is a professional engineer in the state of Texas, an active member of IEEE and the Power System Relaying Committee. Joe can be contacted at jperez@erlphase.com or algorithm@ieee.org.